A Solution Approach for the Joint Order Batching and Picker Routing Problem in Manual Order Picking Systems

André Scholz
Gerhard Wäscher

Otto-von-Guericke University Magdeburg, Germany
Faculty of Economics and Management
- Department of Management Science -

Clausthal, May 2014

Order Picking

Definition

Order picking is a warehouse function dealing with the retrieval of articles from their storage location in order to satisfy a given demand specified by customer orders.

Fig. 1: distribution center of amazon and a picking device (amazon.de, vtechnik.de)

Importance of Order Picking

Of all warehouse operations, order picking is considered to include the most cost-intensive ones.

Between 50% and 65% of the total warehouse operating costs can be attributed to order picking:

- 50% according to Frazelle [2002]
- 55% according to Tompkins et al. [2010]
- 65% according to Coyle et al. [2003].

Content

Content

Content

Problem Description

Given:

- layout of the warehouse (e.g. 2-block layout)
- set of requested articles and their storage location in the warehouse

Question:

How can the sequences, according to which the items have to be picked, be determined such that the tour length is minimized?

2-Block Layout

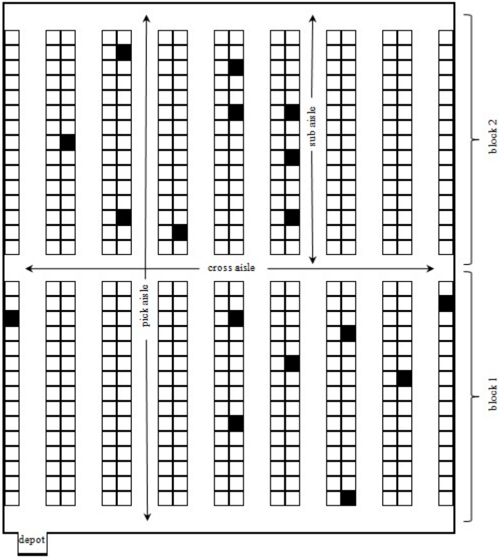

Fig. 2: warehouse with a 2-block layout

Content

S-Shape Policy

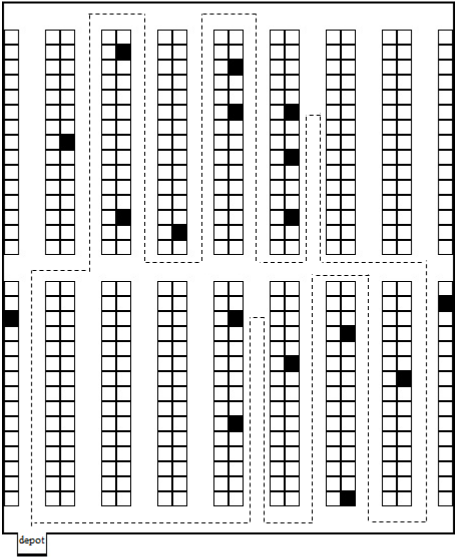

Fig. 3: s-shape policy

Largest Gap Policy

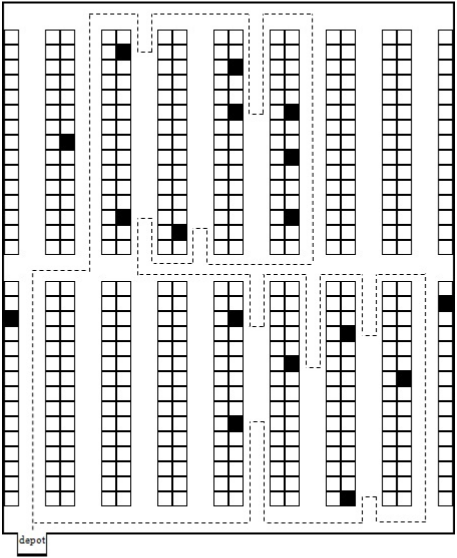

Fig. 4: largest gap policy

Content

Graph Theoretical Representation of the Problem

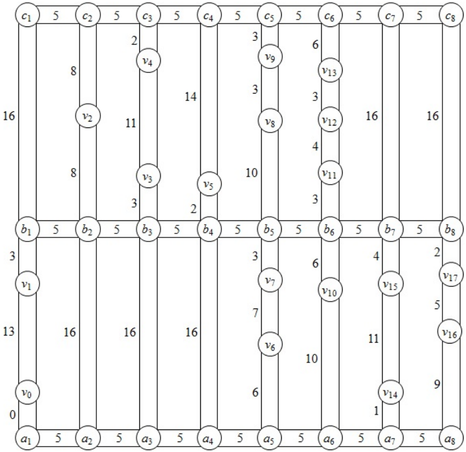

Fig. 5: graph theoretical representation

Possible Configurations

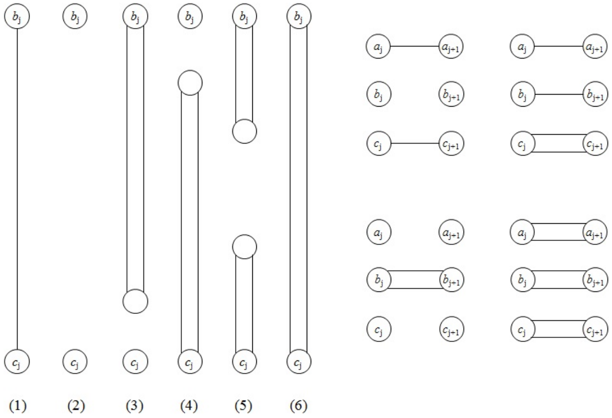

Fig. 6: selection of edges

Adding of Edges

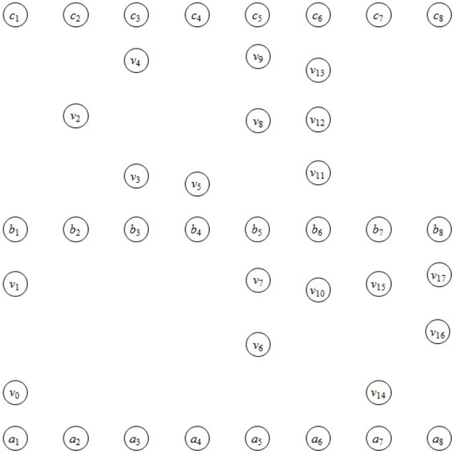

Fig. 7a: start: L_j^- - PTS, aisle $j = 1$ (empty graph)

Adding of Edges

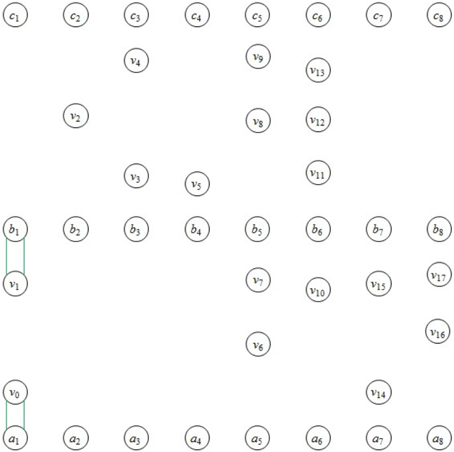

Fig. 7b: first iteration: L_j^{+1} - PTS, aisle $j = 1$

Adding of Edges

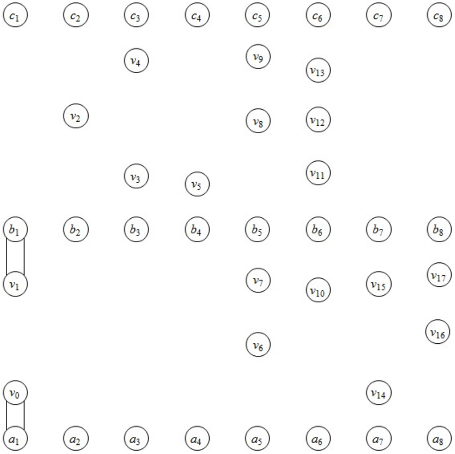

Fig. 7c: second iteration: L_j^{+2} - PTS, aisle $j = 1$

Adding of Edges

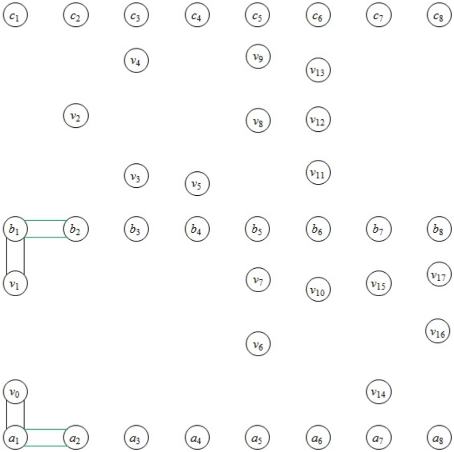

Fig. 7d: third iteration: L_j^- - PTS, aisle $j = 2$

Adding of Edges

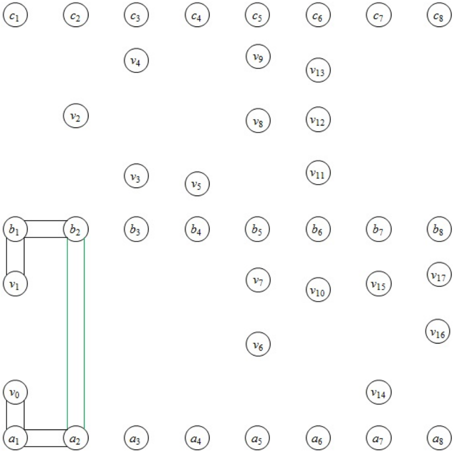

Fig. 7e: fourth iteration: L_j^{+1} - PTS, aisle $j = 2$

Adding of Edges

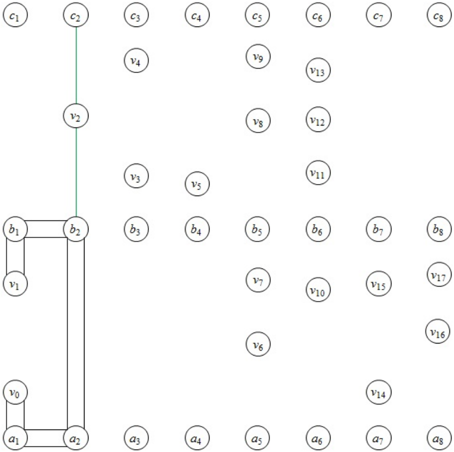

Fig. 7f: fifth iteration: L_j^{+2} - PTS, aisle $j = 2$

Equivalence of Partial Tour Subgraphs

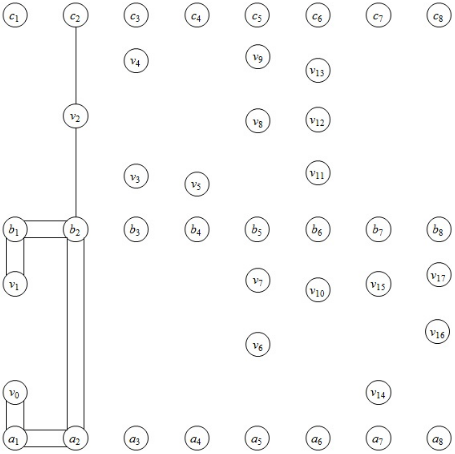

Fig. 8: L_j^{+2} - PTS and a corresponding completion, aisle $j = 2$

Equivalence of Partial Tour Subgraphs

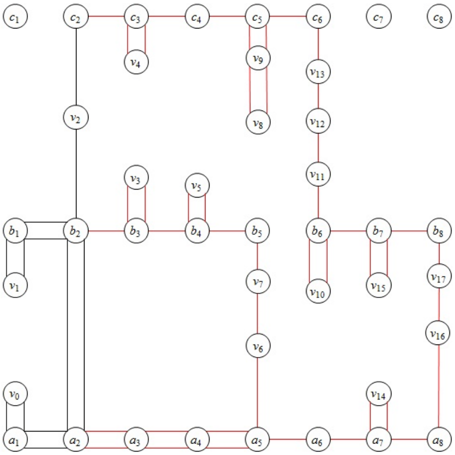

Fig. 8: L_j^{+2} - PTS and a corresponding completion, aisle $j = 2$

Equivalence of Partial Tour Subgraphs

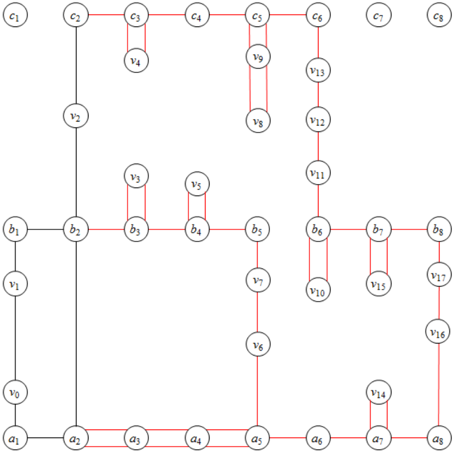

Fig. 8: L_j^{+2} - PTS and a corresponding completion, aisle $j = 2$

Content

Content

Order Batching Problem

Given:

- set of customer orders consisting of requested articles and their storage location in the warehouse
- picking device and its limited capacity
- routing strategy

Question:

How can this set of customer orders be grouped into picking orders such that the total length of all picker tours is minimized?

Joint Order Batching and Picker Routing Problem

Given:

- set of customer orders consisting of requested articles and their storage location in the warehouse
- layout of the warehouse
- picking device and its limited capacity

Question:

How can this set of customer orders be grouped into picking orders and how can the sequences, according to which the items have to be picked, be determined such that the total length of all order picking tours is minimized?

Content

General Principle

- iterated local search (ILS) consists of two alternating phases
- local search phase: improvement of a solution until a local optimum is found
- perturbation phase: modification of a solution in order to overcome local optima

General Principle

- iterated local search (ILS) consists of two alternating phases
- local search phase: improvement of a solution until a local optimum is found
- perturbation phase: modification of a solution in order to overcome local optima

ILS for the joint batching and routing problem

- application of ILS for the order batching problem (Henn et al., 2010)
- routing strategies are used to evaluate solutions in the local search phase and to determine whether a solution passes an acceptance criterion

Pseudocode ILS

Input: problem data, rearrangement parameter λ, threshold parameter μ, time interval t, routing strategy r;

generate initial solution x by applying the FCFS rule;
$x^* := \text{local\_search}(x)$; $x_{\text{inc}} := x^*$;
repeat
    $x := \text{perturbation}(x_{\text{inc}}, \lfloor n^* \cdot \lambda + 1 \rfloor)$;
    $x := \text{local\_search}(x)$;
    if $f_r(x) < f_r(x^*)$ **then**
        $x^* := x$; $x_{\text{inc}} := x$;
    end if
    if no improvement of $f_r(x^*)$ during t **and** $f_r(x) - f_r(x^*) < \mu \cdot f_r(x^*)$ **then**
        $x_{\text{inc}} := x$;
    end if
until termination condition is met

Content

Setup

Instances

- number of pick aisles $m \in \{10, 30\}$
- number of storage locations per sub aisle $= 50$
- number of orders $N \in \{20, 40, 60, 80\}$
- number of requested articles per order $\sim \mathrm{U}\{5, \ldots, 25\}$
- capacity of the picking device $C \in \{30, 45, 60, 75\}$
- 50 instances per problem class

Iterated local search

- rearrangement parameter $\lambda = 0.3$
- threshold parameter $\mu = 0.05$
- time limit $T = 3 \cdot N$
- time interval $t = \frac{T}{10}$

Results (1)

Table 1: number of iterations in the ILS algorithm ($C = 30$)

$(\mathbf{m}, \mathbf{N})$	**SS**	**LG**	**Exact**
(10, 20)	9220	3822	181
(10, 40)	3145	1058	60
(10, 60)	1251	455	21
(10, 80)	717	206	7
(30, 20)	3481	1861	81
(30, 40)	1117	543	20
(30, 60)	376	174	6
(30, 80)	209	97	3

Results (2)

Table 2: deviation from objective function value obtained by ILS + optimal routing [%]

$(\mathbf{m},\mathbf{N},\mathbf{C})$	**ILS**	
	SS	LG
(10, 20, 30)	20.28	17.90
(10, 20, 45)	14.93	20.21
(10, 20, 60)	12.21	22.72
(10, 20, 75)	9.93	24.68
(10, 80, 30)	18.31	17.49
(10, 80, 45)	13.30	19.38
(10, 80, 60)	10.23	21.40
(10, 80, 75)	7.85	23.56
(30, 80, 30)	23.37	24.62
(30, 80, 45)	22.60	23.35
(30, 80, 60)	21.87	23.73
(30, 80, 75)	20.39	23.68
average	16.27	21.89

Results (3)

Table 3: deviation after application of the exact routing algorithm [%]

$(\mathbf{m}, \mathbf{N}, \mathbf{C})$	**ILS**	
	SS	LG
(10, 20, 30)	1.03	0.92
(10, 20, 45)	2.45	1.64
(10, 20, 60)	2.75	2.06
(10, 20, 75)	3.06	1.87
(10, 80, 30)	0.97	0.80
(10, 80, 45)	2.70	1.65
(10, 80, 60)	3.26	1.94
(10, 80, 75)	3.04	1.94
(30, 80, 30)	0.04	0.27
(30, 80, 45)	1.05	1.40
(30, 80, 60)	2.17	2.12
(30, 80, 75)	2.50	2.18
average	2.40	1.60

Results (4)

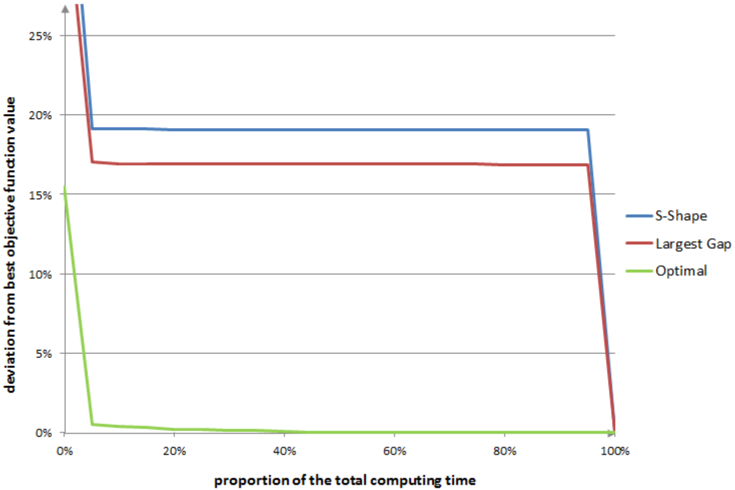

Fig. 9: development of the solution quality over time ($m = 10$, $N = 20$, $C = 30$)

Content

Conclusion and Outlook

Summary

- optimization of the routes is pivotal and leads to significant savings
- application of the exact routing algorithm in the ILS results in an improvement of the solution quality without increasing the computing time

Conclusion and Outlook

Summary

- optimization of the routes is pivotal and leads to significant savings
- application of the exact routing algorithm in the ILS results in an improvement of the solution quality without increasing the computing time

Further Research

- solution of problems with class-based storage assignment
- design of a VRP-based model formulation
- consideration of picker blocking aspects

Thank you for your attention!

Concept and Sets

Model formulation provided by Gademann and van de Velde [2005].

Concept

Generate all feasible batches and choose some of these batches in such a way that each order is contained in (at least) one batch and the total travel time is minimized.

Sets

J: set of customer orders

F: set of all feasible batches
(A batch is feasible if the number of contained articles is not larger than the capacity of the picking device.)

Constants and Variables

Constants

d_f: minimal length of a tour which includes all articles of customer orders contained in batch f ($f \in F$)

$$a_{fj} = \begin{cases} 1, & \text{if customer order } j \text{ is contained in batch } f \\ 0, & \text{otherwise} \end{cases} \quad (f \in F, j \in J)$$

Variables

$$x_f = \begin{cases} 1, & \text{if batch } f \text{ is chosen} \\ 0, & \text{otherwise} \end{cases} \quad (f \in F)$$

Model Formulation

$$\min \sum_{f \in F} d_f \cdot x_f \tag{4.1}$$

$$\sum_{f \in F} a_{fj} \cdot x_f = 1 \quad \forall j \in J \tag{4.2}$$

$$x_f \in \{0, 1\} \quad \forall f \in F \tag{4.3}$$

NP-hardness of the joint problem

Decision variant π of the problem

Does there exist a partition of the N orders into batches, with each batch containing no more than C articles, such that the total distance to be traveled is smaller than or equal to a given constant α?

Proof for NP-completeness of π

1) $\pi \in$ NP: ✓ (certificate: solution with objective function value $\leq \alpha$)
2) $\pi \in$ NP-hard: (polynomial) reduction from the order batching problem defined by Gademann & van de Velde (2005)

NP-hardness of the joint problem (2)

Order batching problem π^G in Gademann & van de Velde (2005)

- layout of the warehouse: single block layout
- capacity C^G expressed in number of orders

Reduction to problem π

Let n_i^G $(i = 1, \ldots, N)$ be the number of articles contained in order i of a problem π^G and let η_i be an article contained in order i.

For any instance of π^G, define a problem π with the same layout and identical customer orders.

To each order i in π, article η_i is added $\max\left\{n_j^G \mid j = 1, \ldots, N\right\} - n_i^G$ times.

Set the capacity C in π (expressed in number of articles) to $C = \max\left\{n_j^G \mid j = 1, \ldots, N\right\} \cdot C^G$.

Combined^{+} Heuristic

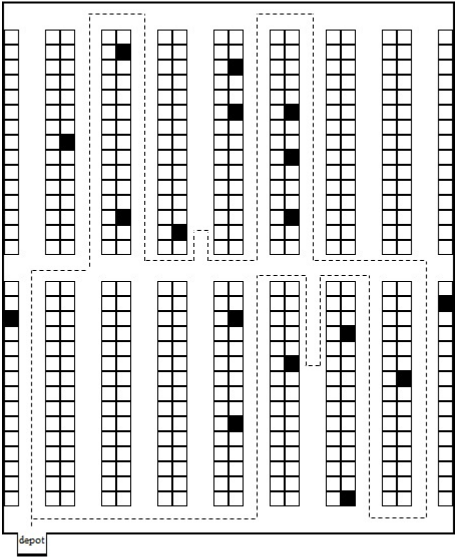

Fig. 10: combined^{+} heuristic

Equivalence of L_j - PTS

Theorem (Roodbergen & de Koster, 2001)

Two L_j - PTS T_j^1 and T_j^2 are equivalent if:

a) a_j, b_j and c_j have the same degree parity (i.e. even, odd, zero) in both

b) excluding vertices with zero degree, both T_j^1 and T_j^2 have either
- no connected component
- a single connected component containing at least one of a_j, b_j and c_j
- two connected components with in each component at least one of a_j, b_j and c_j and each of a_j, b_j and c_j contained in at most one component
- three connected components with a_j, b_j and c_j each in a different one

c) the distribution of a_j, b_j and c_j over the various components is the same for both subgraphs

Corollary (Roodbergen & de Koster, 2001)

There are only 25 different equivalence classes.

General Concept

Exact Solution Approach

- time-complexity function is linear in the number of pick aisles and the number of requested articles, but contains a large coefficient
- coefficient originates from adding the configurations to a PTS of each equivalence class

  $\Longrightarrow$ consideration of up to $25 \cdot 26 = 650$ subgraphs per pick aisle

Heuristic Solution Approach (denoted by BCO)

- deletion of all but the shortest subgraph after each change of a pick aisle, i.e. after each transition from L_j^{+2} - PTS to L_{j+1}^{-} - PTS
- configuration (6) is disregarded

Feasibility

- only eight equivalence classes of $L^{+2}_{\widetilde{m}}$ - PTS (where $\widetilde{m}$ denotes the rightmost pick aisle containing at least one requested article) correspond to feasible order picking tours
- two of which may lead to an infeasible solution if they occur as $L^{+1}_{\widetilde{m}}$ - PTS

  $\Longrightarrow$ generation of a feasible solution cannot be guaranteed

Modification

At each step, an additional PTS is considered if the shortest PTS does not belong to a class that guarantees the generation of a feasible order picking tour.

Feasibility (2)

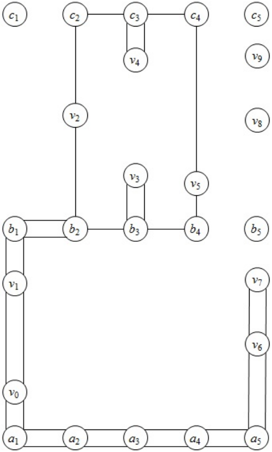

Fig. 11a: $L_{\tilde{m}}^{+1}$ - PTS

Feasibility (2)

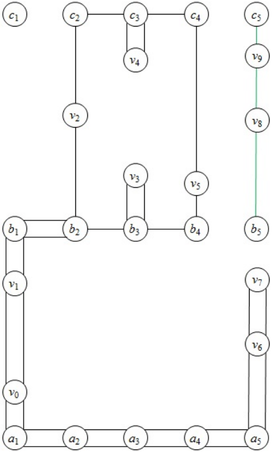

Fig. 11b: $L_{\tilde{m}}^{+2}$ - PTS

Feasibility (2)

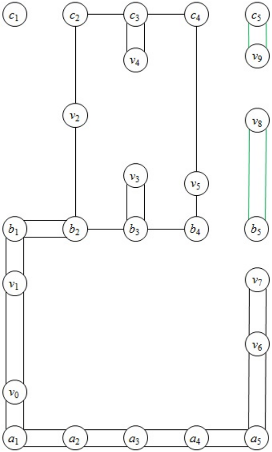

Fig. 11c: $L_{\tilde{m}}^{+2}$ - PTS

Solution Quality

- deletion of subgraphs may lead to a poor solution quality
- poor decisions at early stages of the algorithm caused by edge configurations having a large impact on the sum of edge weights of a subgraph
- if the number of requested articles per sub aisle is small, configurations (3) to (5) will be preferred resulting in unconnected subgraphs

  $\Longrightarrow$ at a later stage, configuration (1) has to be added to connect the subgraph (regardless of the impact on the tour length)

Modification

Let m be the number of pick aisles and $p \in [0, 1]$.

- each aisle $j \leq \lceil p \cdot m \rceil$: application of exact solution approach
- each aisle $j > \lceil p \cdot m \rceil$: application of heuristic approach

Solution Quality (2)

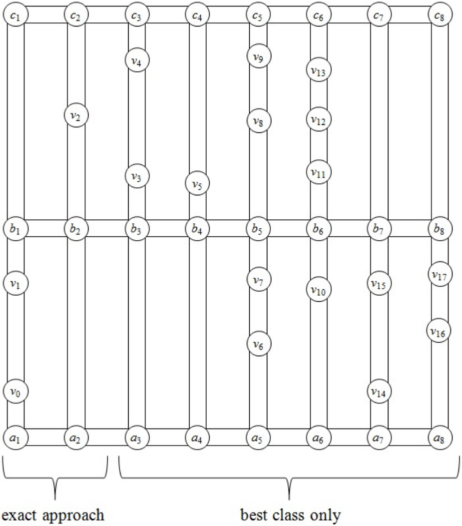

Fig. 12: BCO with $m = 8$ and $p = 0.25$

Setup

- depot is positioned in front of the leftmost pick aisle
- number of pick aisles $m \in \{10, 20, 30\}$
- number of storage locations per sub aisle $= 50$
- length of a storage location $= 1$
- distance between two adjacent pick aisles $= 5$
- number of requested articles $n \in \{30, 45, 60, 75\}$
- 100 instances per problem class

Results

Table 4: deviation from optimal objective function value [%]

(m,n)	**SS**	**LG**	**C$^+$**	**BCO**
(10, 30)	20.2	19.3	6.8	**4.0**
(10, 45)	15.2	22.2	5.1	**3.2**
(10, 60)	9.6	23.4	3.7	**2.3**
(10, 75)	8.9	26.5	3.2	**2.9**
(20, 30)	27.2	22.3	7.2	**5.8**
(20, 45)	24.4	22.1	6.9	**4.5**
(20, 60)	20.8	23.7	6.4	**3.1**
(20, 75)	18.4	23.5	6.6	**2.7**
(30, 30)	27.4	27.7	**7.0**	7.6
(30, 45)	26.3	26.3	7.3	**5.4**
(30, 60)	25.0	24.3	7.0	**4.3**
(30, 75)	23.9	25.1	7.0	**3.4**
average	20.6	23.9	6.2	**4.1**

Local Search Phase

- first improvement strategy
- improvement of an initial solution by applying moves according to two neighborhood structures

Neighborhood structures

$\mathcal{N}_1$: exchange two customer orders of different batches;

$\mathcal{N}_2$: move a customer order to a different batch

Perturbation Phase

- modification of an initial solution x by executing γ rearrangements

A rearrangement in the perturbation phase

1. choose two batches k and l from x randomly
2. choose $q > 0$ and smaller than half of the number of orders in k and l
3. remove the first q orders from k and l
4. insert the removed orders from k into l (and the removed orders from l into k) as long as the capacity constraint is not violated
5. assign the remaining orders to a new batch

Results

Table 2: deviation from objective function value obtained by ILS + optimal routing [%]

$(\mathbf{m}, \mathbf{N}, \mathbf{C})$	**ILP**	**ILS**			
		SS	LG	C^+	BCO
(10, 20, 30)	-0.23	1.03	0.92	0.72	0.96
(10, 20, 45)	-0.25	2.45	1.64	0.95	0.66
(10, 20, 60)	-	2.75	2.06	1.20	0.63
(10, 20, 75)	-	3.06	1.87	1.12	0.49
(10, 80, 30)	-	0.97	0.80	0.43	1.18
(10, 80, 45)	-	2.70	1.65	1.09	1.18
(10, 80, 60)	-	3.26	1.94	1.37	1.33
(10, 80, 75)	-	3.04	1.94	1.26	1.32
(30, 80, 30)	-	0.04	0.27	0.38	1.72
(30, 80, 45)	-	1.05	1.40	0.68	1.88
(30, 80, 60)	-	2.17	2.12	1.49	2.30
(30, 80, 75)	-	2.50	2.18	1.48	2.34
average	-	2.40	1.60	0.97	0.98

References

Albareda-Sambola, M. et al. (2009): Variable Neighborhood Search for Order Batching in a Warehouse. In: *Asia-Pacific Journal of Operational Research* 26, 655-683.

de Koster, R. et al. (1999): Efficient Orderbatching Methods in Warehouses. In: *International Journal of Production Research* 37, 1479-1504.

de Koster, R. et al. (2007): Design and Control of Warehouse Order Picking: A Literature Review. In: *European Journal of Operational Research* 182, 481-501.

Hall, R.W. (1993): Distance Approximations for Routing Manual Pickers in a Warehouse. In: *IIE Transactions* 25, 76-87.

Henn, S. et al.(2010): Metaheuristics for the Order Batching Problem in Manual Order Picking Systems. In: *BuR - Business Research* 3, 82-105.

Henn, S. et al. (2012): Order Batching in Order Picking Warehouses: A Survey of Solution Approaches. In: Manzini R., Hrsg., Warehousing in the Global Supply Chain: Advanced Tools and Applications for Storage Systems, Springer-Verlag London, 105-137.

References

Henn, S; Wäscher, G.: Tabu Search Heuristics for the Order Batching Problem in Manual Order Picking Systems. In: *European Journal of Operational Research* 222, 484-494.

Gademann, N.; van de Velde, S. (2005): Order Batching to Minimize Total Travel Time. In: *IIE Transactions* 37, 63-75.

Goetschalckx, M.; Ratliff, H.D. (1988): Order Picking In An Aisle. In: *IIE Transactions* 20, 53-62.

Gibson, R.D.; Sharp, G.P. (1992): Order Batching Procedures. In: *European Journal of Operational Research* 58, 57-67.

Ratliff, H.D.; Rosenthal, A.S. (1983): Order-Picking in a Rectangular Warehouse: A Solvable Case of the Traveling Salesman Problem. In: *Operations Research* 31, 507-521.

Roodbergen, K.J. (2001): Layout and Routing Methods for Warehouses, Trial, Rotterdam.